A New Approach to the Analysis of Turbulent Free Convection Heat Transfer

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The principle of surface renewal is utilized in the analysis of fully turbulent free convection. The present analysis together with a previous surface renewal based analysis of combined forced and free convection associated with supercritical fluids demonstrate the potential usefulness of the surface renewal approach in modelling buoyancy influenced turbulent convection processes.

NOTATION

g	Gravitational acceleration
Gr_x	Grashof number ($\equiv g\beta x^3(T_0 - T_\infty)/\nu^2$)
Nu_x	Nusselt number ($\equiv hx/k$)
Pr	Prandtl number ($\equiv \nu/\alpha$)
S	Mean frequency of thermal waves near wall
u	Instantaneous velocity profile
$\bar{u}$	Mean velocity profile
$\bar{u}_p$	Peak mean velocity
U_i	Velocity at first instant of renewal
U^*	Friction velocity ($\equiv \sqrt{(\tau_0/\rho)}$)
T	Instantaneous temperature profile
$\bar{T}$	Mean temperature profile
T_0	Wall temperature
T_∞	Free stream temperature
T_b	Bulk stream temperature within thermal boundary layer
T_i	Temperature at first instant of renewal
x	Axial coordinate
y	Distance from wall
y_p	Location of peak velocity
y_p^+	Dimensionless distance ($\equiv y_p U^*/\nu$)
α	Thermal diffusivity
β	Volumetric expansion coefficient
μ	Dynamic viscosity
ν	Kinematic viscosity
ρ	Density
θ	Contact time
ϕ	Contact time distribution
τ_{0x}	Wall shear stress
τ	Mean period of time burst process
ψ_i	Dimensionless temperature ($\equiv T_i - T_0)/(T_\infty - T_0)$

INTRODUCTION

Considerable attention has been given to the problem of turbulent convection heat transfer under conditions of variable physical properties. The classical eddy diffusivity approach to variable property flows has produced mixed results, with some of the greatest difficulties encountered in dealing with combined forced and natural convection systems. In particular, experiments for vertical flow of supercritical fluids (**1**), gases (**2**), and liquid metals (**3**) reveal deterioration in the heat transfer for upflow and enhancement for downflow. But, predictions using the eddy diffusivity approach follow the opposite trend (**2**), (**4**). Of course large variation in eddy diffusivity values and their dependence on heat flux when buoyancy effects are significant make it very difficult to apply classical turbulent heat transfer models to these types of flow.

An alternative approach to the analysis of turbulent convection heat transfer has evolved over the past few years which produces predictions for combined forced/natural convection flows which are in basic agreement with experimental data (**4**), (**5**). In this surface renewal approach to turbulent convection heat transfer, the unsteady burst phenomenon associated with wall turbulence is modelled. The underlying principle of surface renewal is in basic agreement with recent flow visualization and anemometry studies (**6**), (**7**) which indicate the existence of strong eddy motion very near the wall for conditions of turbulent flow. In order to establish more clearly the potential of this approach in analysing the effects of buoyancy on turbulent convection processes, attention is focused in this paper on fully turbulent natural convection flow over a vertical flat plate.

Until recently, no truly adequate analysis has been available for turbulent natural convection heat transfer over vertical surfaces. Early preliminary analyses by Eckert and Jackson (**8**), Bayley (**9**), and Kato *et al.* (**10**) which were based on the assumed similarity between forced and free convection provided reasonable predictions for the rate of heat transfer, but have not been found to be consistent with data for the mean velocity profile (**11**), (**12**). More successful analyses have been recently developed by Mason and Seban (**13**) and Cebeci and Khattab (**14**) in which the continuity, momentum, and energy equations were integrated numerically. The turbulent viscosity ν_t was expressed in terms of the turbulent kinetic energy by Mason and Seban, whereas Cebeci and Khattab utilized a damping factor type model. These approaches both require a minimum of three empirical inputs.

Recent experimental work reported by Lock and Trotter (**15**), Akins (**16**), and Black and Norris (**17**) indicate that an unsteady viscous sublayer is also present in turbulent free convection. The anemometer signals obtained by wall probes (**15**), (**16**) and measurements made using a differential interferometer (**17**) possess the same basic fluctuating characteristic for turbulent free convection as forced convection. This evidence provides further impetus for the application of the principle of surface renewal to turbulent natural convection.

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ANALYSIS

This analysis will be restricted to incompressible, moderate Prandtl number, Newtonian, fully turbulent flow over a vertical flat plate with constant surface temperature. During the brief residency of eddies in the wall region, unsteady momentum and energy transport occur simultaneously. The instantaneous energy transport to individual elements of fluid residing within the wall region is approximated by

$$\frac{\partial T}{\partial \theta} = \alpha \frac{\partial^2 T}{\partial y^2} \qquad (1)$$

with initial-boundary conditions of the form $T(0, y) = T_i$, $T(\theta, 0) = T_0$, and $T(\theta, \infty) = T_i$. The inclusion of the buoyancy force in the analysis for momentum transfer leads to a momentum equation of the form

$$\frac{\partial u}{\partial \theta} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) \qquad (2)$$

The appropriate initial and boundary conditions are $u(0, y) = U_i$, $u(\theta, 0) = 0$, and $u(\theta, \infty) =$ finite. The evaluation of the parameters T_i and U_i will be discussed shortly.

These equations are transformed into the mean domain by the use of the random contact time distribution function proposed by Danckwerts (**18**),

$$\phi(\theta) = \frac{1}{\tau} \exp\left(-\frac{\theta}{\tau}\right) \qquad (3)$$

where τ is the mean residence time. The solutions to these equations for the mean temperature and velocity profiles take the forms (**19**)

$$\frac{\bar{T} - T_\infty}{T_0 - T_\infty} = \psi_i \exp\left(-\frac{y}{\sqrt{(\alpha\tau)}}\right) + 1 - \psi_i \qquad (4)$$

$$\bar{u} = U_i\left\{1 - \exp\left(-\frac{y}{\sqrt{(\nu\tau)}}\right)\right\} + g\beta\tau(T_0 - T_\infty) \times \left[(1 - \psi_i)\left\{1 - \exp\left(-\frac{y}{\sqrt{(\nu\tau)}}\right)\right\} + \frac{\psi_i}{Pr - 1}\left\{1 - \exp\left(-\frac{y}{\sqrt{(\alpha\tau)}}\right)\right\}\right] \qquad (5)$$

for $Pr \neq 1$ where $\psi_i = (T_i - T_0)/(T_\infty - T_0)$. Expressions therefore can be written for the mean wall shear stress and Nusselt number of the forms

$$\frac{\tau_{0x}}{\rho} = g\beta\sqrt{(\nu\tau)}(T_0 - T_\infty)\left(1 - \psi_i + \frac{\psi_i\sqrt{(Pr)}}{Pr - 1}\right) + \frac{U_i\sqrt{\nu}}{\sqrt{\tau}} \qquad (6)$$

and

$$Nu_x = \psi_i \frac{x}{\sqrt{(\nu\tau)}} \qquad (7)$$

These expressions reduce to previous results for forced convection (**20**) when the volumetric expansion coefficient is zero.

Similar to the analysis of turbulent forced convection (**20**), T_i is set equal to the bulk mean fluid temperature T_b across the thermal boundary layer as a first approximation. However, the more complex non-monotonic behaviour of the mean velocity profile for turbulent free convection makes the parameter U_i more difficult to approximate. Therefore this parameter will not be specified *a priori*, such that the resulting relationships for mean heat and momentum transfer proposed in this analysis involve two unspecified modelling parameters, U_i and τ. Two empirical inputs will be required to evaluate these parameters. In this analysis, experimental measurements of the mean wall shear stress and the velocity at a point within the wall region will be used for this purpose.

APPLICATION-TURBULENT FREE CONVECTION IN AIR

The most extensive measurements within the wall region for turbulent free convection have been reported by Cheesewright (**12**) for air. These experimental data for the mean velocity profile near the wall provide a means of evaluating the mean wall shear stress in turbulent free natural convection. These results are well correlated by an empirical equation of the form

$$\tau_{0x} = 0{\cdot}167\rho \frac{\nu^2}{x^2} Gr_x^{0{\cdot}806} \qquad (8)$$

This information will be utilized as one of the empirical inputs.

Because the surface renewal model is restricted to the wall region, the velocity at any point within this region could serve as the second empirical input. However, the peak velocity most conveniently serves this purpose and will be utilized in this analysis. Based on the work of Cheesewright, the peak velocity $\bar{u}_p$ has been found to be located at $y = y_p = 0{\cdot}3xGr_x^{0{\cdot}1}$ with the value of $\bar{u}_p = 0{\cdot}3U^* = 0{\cdot}3\{gx(T_0 - T_\infty)\}^{1/2}$ for all values of Gr_x in the fully turbulent region. The dimensionless distance y_p^+ is equal to 16·5 and 40 for Gr_x equal to 3×10^{10} and 6×10^{11}, respectively. Consequently, the peak velocity occurs within the wall region in which eq. (5) is applicable.

The present analysis, as well as previous analyses of turbulent natural convection, is restricted to fully turbulent conditions which occur for flow over a vertical plate at Gr_x somewhat above 10^{10}. In this regard, the empirical inputs $\bar{u}_p$ and τ_{0x} were procured in the fully turbulent domain ($Gr_x \geqslant 3 \times 10^{10}$). These key inputs cannot be assumed to be appropriate for lower values of Gr_x.

An expression can be written for U_i as functions of τ and τ_{0x} from eq. (6) as (with $T_i = T_b$)

$$U_i = \rho g\beta\tau\left\{(1 - \psi_b) + \frac{\psi_b}{\sqrt{(Pr + 1)}}\right\}(T_\infty + T_0) + \tau_{0x}\frac{\sqrt{(\nu\tau)}}{\mu} \qquad (9)$$

The substitution of eq. (9) into eq. (5) results in a relationship for the mean velocity as a function of the single unspecified variable τ. By imposing the condition $\bar{u} = \bar{u}_p$ at $y = y_p$, an implicit equation for τ results. The resulting equation has been solved by iteration for τ (**17**). These predictions for τ are shown in Fig. 1. The mean residence time is seen to gradually fall to a constant value for Gr_x of the order of 2×10^{11}. Although no experimental measurements of τ have been published for turbulent

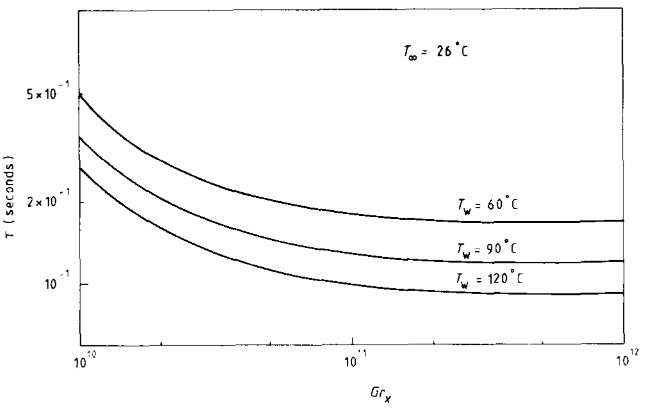

Fig. 1. Prediction of mean residence time for air

free convection on a vertical flat plate, data for periodicity of thermal waves very near the wall (1/*S*) associated with inclined surfaces have recently been presented by Black and Norris (**17**). Data for *S* and Nusselt number are reported in reference (**17**) for a wall temperature of 140° C and air temperature of 24° C, and inclinations from the vertical of 45°, 60°, 70°, and 80°. The measurements for 1/*S* were found to follow the same trends as τ predicted in the present study.

The parameter U_i is specified by substituting the value of τ into eq. (9). Calculations for this parameter are given in Fig. 2. These calculations are seen to increase with Gr_x in the fully turbulent region.

Based on this evaluation of the two modelling parameters U_i and τ, predictions for the mean temperature and velocity profiles and Nusselt number are given by eqs. (4), (5), and (7). Predictions for velocity profiles near the wall are shown in Fig. 2 along with experimental data by Cheesewright for three different values of Gr_x. The agreement between theory and experiment in the region between the wall and y_p is excellent. Beyond the wall region, the agreement degenerates as expected. It should be noted that the empirical input away from the wall only involves the magnitude of $\bar{u}$ and $y = y_p$. The fact that this analysis gives rise to peak velocities at these three values of Gr_x in the close proximity of the actual peak further reinforces the model.

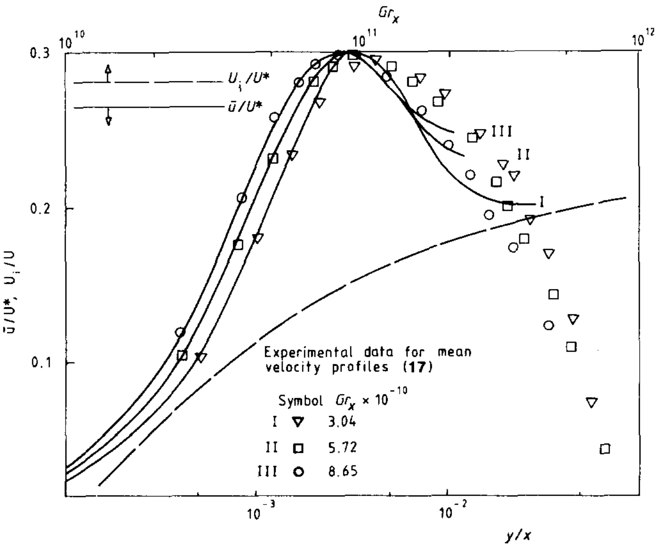

Fig. 2. Predictions for U_i and *u* and comparison with experimental data for *u*

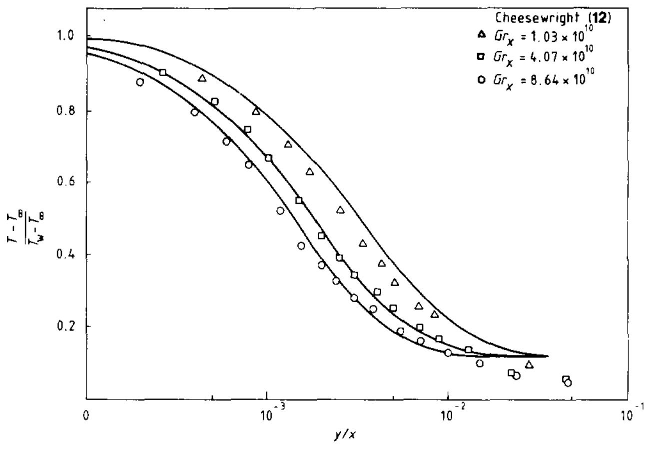

Fig. 3. Comparison of predicted temperature profile with experimental data

Predictions for the temperature distributions are compared with the experimental data in Fig. 3. The agreement between theory and data is exceptional for higher Gr_x but deteriorates somewhat for $Gr_x = 1{\cdot}03 \times 10^{10}$, which apparently lies in the transition region.

Predictions for Nu_x are compared with various experimental data (**12**), (**21**), (**22**) and other theoretical curves in Fig. 4. The present analysis is in excellent agreement with experimental data for Gr_x greater than 3×10^{10}.

CONCLUSION

The key aspect of the proposed surface renewal based formulation for fully turbulent free convection is the inclusion of the buoyancy body force in the instantaneous momentum equation. As a consequence of the effects of this force on the momentum transfer, a non-monotonic mean velocity profile is produced. Based on the surface renewal model, predictions have been developed for U_i, τ, $\bar{u}$, $\bar{T}$, and Nu_x for turbulent free convection on a vertical flat plate. Although experimental data are not available for U_i and τ, the excellent agreement between theory and data for $\bar{u}$, $\bar{T}$, and Nu_x for air serve as a strong test of the model. The basic agreement in the

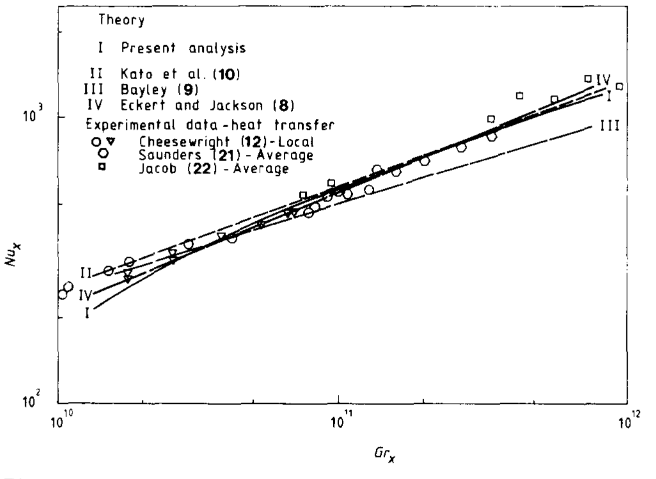

Fig. 4. Predictions for mean residence time and Nusselt number and comparison with heat transfer data

predictions for τ with data trends for turbulent free convection on inclined surfaces **(15)** also serves to reinforce the analysis.

The present analysis together with the previous surface renewal based analysis of combined forced and free convection associated with supercritical fluids **(4)**, **(5)** demonstrate the potential usefulness of the surface renewal approach in modelling buoyancy influenced turbulent convection processes.

It should also be recalled that the present analysis only involves two empirical inputs, the wall shear stress τ_{0x} and the peak velocity $\bar{u}_p$. The first of these empirical inputs can be eliminated when more data become available for the mean periodicity of wall turbulence τ. The elimination of the second will require the interfacing of the surface renewal model of wall turbulence with classical concepts within the turbulent core. This advancement is expected to occur within the next year or so.

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